

The Islamic University Of Gaza
Department of Mathematics
Calculus A (MathA1401)

Date: 13/1/2003

Final Exam

Time: Two Hours

	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7	Total
	10	18	14	14	14	10	20	100

Answer the following questions:

Q.1 Choose the correct answer for each of the following:

(i) Let c be an interior point of the domain of a function $f(x)$. If $f'(c) = 0$ or $f'(c)$ is undefined then c is called

(a) a critical point (b) an inflection point (c) a point of discontinuity (d) a midpoint.

(ii) If $f''(x) > 0$ on the interval (a, b) , then $f'(x)$ is

(a) decreasing (b) concave up (c) increasing (d) concave down.

(iii) If $f(x)$ is differentiable on the interval $[a, b]$, then $f(x)$ satisfies the hypotheses of the

(a) sandwich theorem (b) mean value theorem (c) Rolle's theorem (d) all of them.

(iv) If the average of $f(x)$ on the interval $[1, 4]$ is $av(f) = 9$, then

(a) $\int_1^4 f(x)dx = 3$ (b) $\int_1^4 f(x)dx = 4$ (c) $\int_1^4 f(x)dx = 9$ (d) $\int_1^4 f(x)dx = 27$

(v) The linearization of $f(x) = x^3 - 2x + 3$ at $x = 0$ is

(a) $L(x) = 2x - 3$ (b) $L(x) = 3x - 2$ (c) $L(x) = 2 - 3x$ (d) $L(x) = 3 - 2x$

Q.2 (a) Find $\frac{dy}{dx}$ if $y = \cot \sqrt{2x^2 + 1}$.

(b) Find $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{\sqrt{x + 2} - 2}$

(c) Let $f(x) = \begin{cases} 3x^2 - 1, & x \leq 1 \\ kx, & x > 1. \end{cases}$ For what value of k is $f(x)$ continuous at $x = 1$.

Q.3 (a) Find a $\delta > 0$ such that for all x
 $0 < |x - 9| < \delta \Rightarrow |\sqrt{x - 5} - 2| < 1.$

(b) Consider a region in the first quadrant enclosed between the x -axis and the parabola $y = 4 - x^2$. A rectangle is to be inscribed in this region. What is the largest area the rectangle can have?

Q.4 Evaluate the following integrals

(a) $\int_1^3 x(x^2 - 1)^{2/3} dx$

(b) $\int \frac{\sin(2x)}{(3 + \cos x)^3} dx$

Q.5 (a) A region , (see the figure below), bounded by $y = -x$, $y = 2$, and $y = \sqrt{x}$ is revolved about the x -axis to generate a solid. Find the volume of this solid.

(b) Find the area of the surface generated by revolving the curve $y = \sqrt{4 - x^2}$, $0 \leq x \leq 1$, about the x -axis.

Q.6 Use the upper Riemann sum to evaluate the integral $\int_0^3 (x^2 + 3)dx$.
(**Hint:** Partition $[0, 3]$ into n equal subintervals and write out the approximation sum of circumscribed rectangles.)

Q.7 Let $f(x) = \frac{x^2}{x^2 - 1}$.

- (a) Find the intervals on which $f(x)$ is decreasing, increasing, concave up, and concave down.
- (b) Find the location of all extreme values and inflection points of $f(x)$, if they exist.
- (c) Find all the asymptotes of $f(x)$.
- (d) Graph $y = f(x)$.